



# Short collusion-secure fingerprint codes against three pirates

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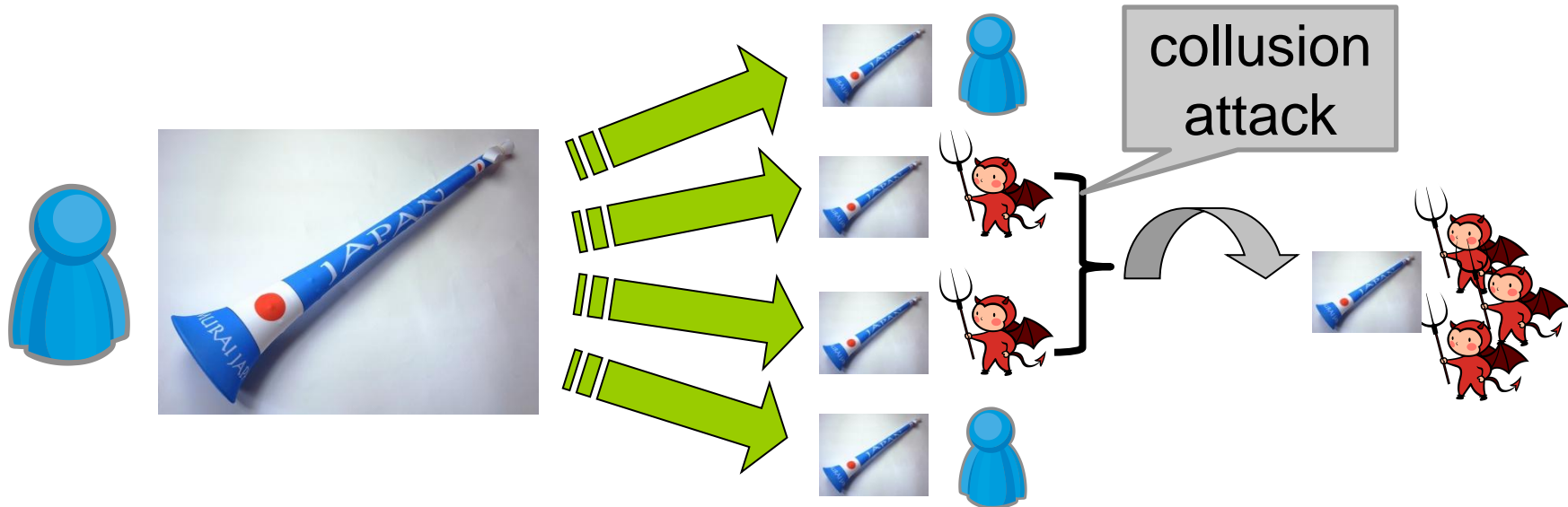
RCIS, AIST, Japan

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# Outline

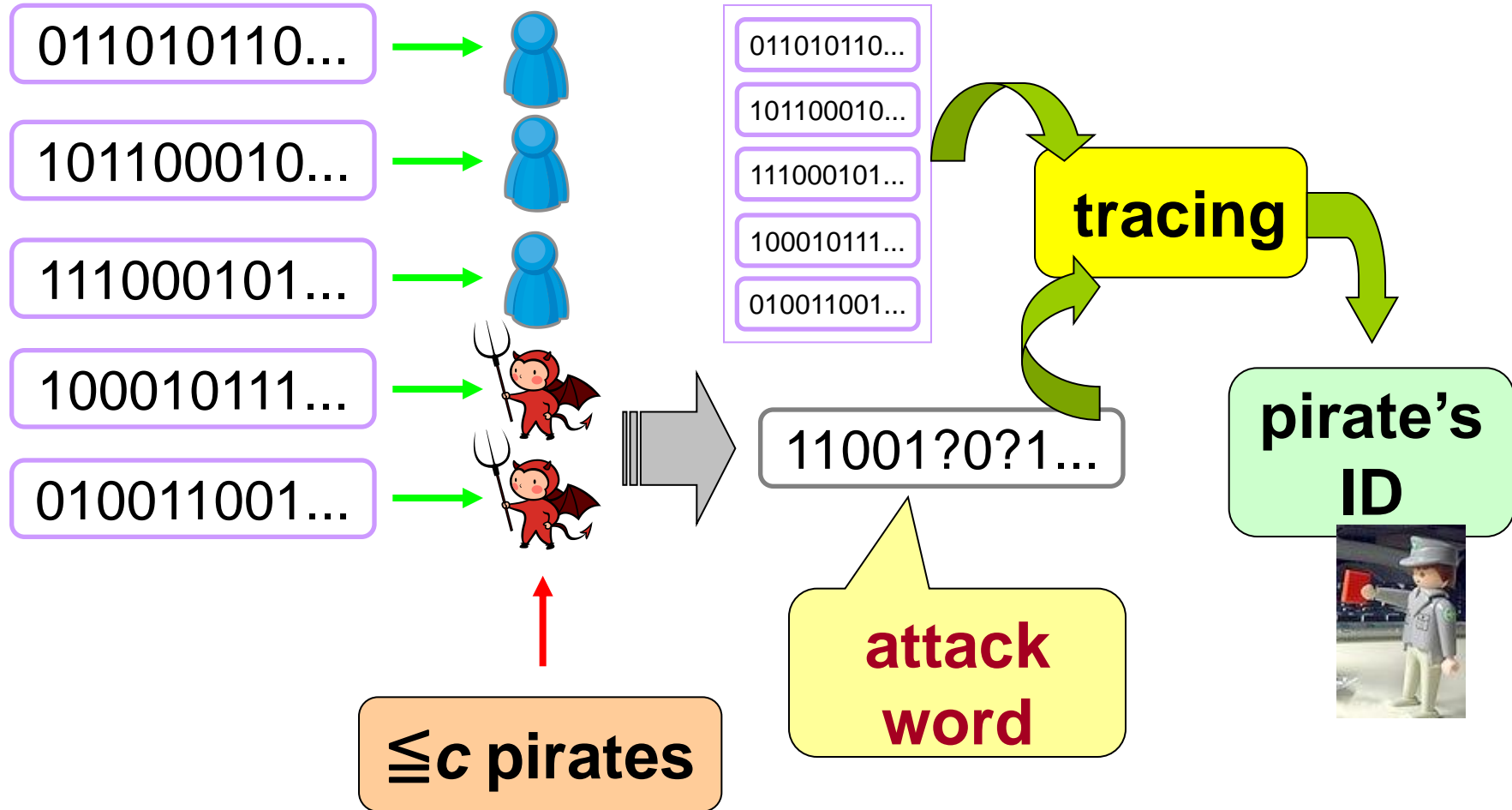
- Problem setting
- Preceding works
- Our contribution: Short 3-secure code
  - E.g., 100 users, 135 bits → 0.9% error
  - Codeword generation (not new)
  - Tracing algorithm (key point)
- Comparison of code lengths
- Observation for speedup of tracing

# Problem



- How to prevent illegal redistribution of copied digital content?
  - How to determine the “pirate”?

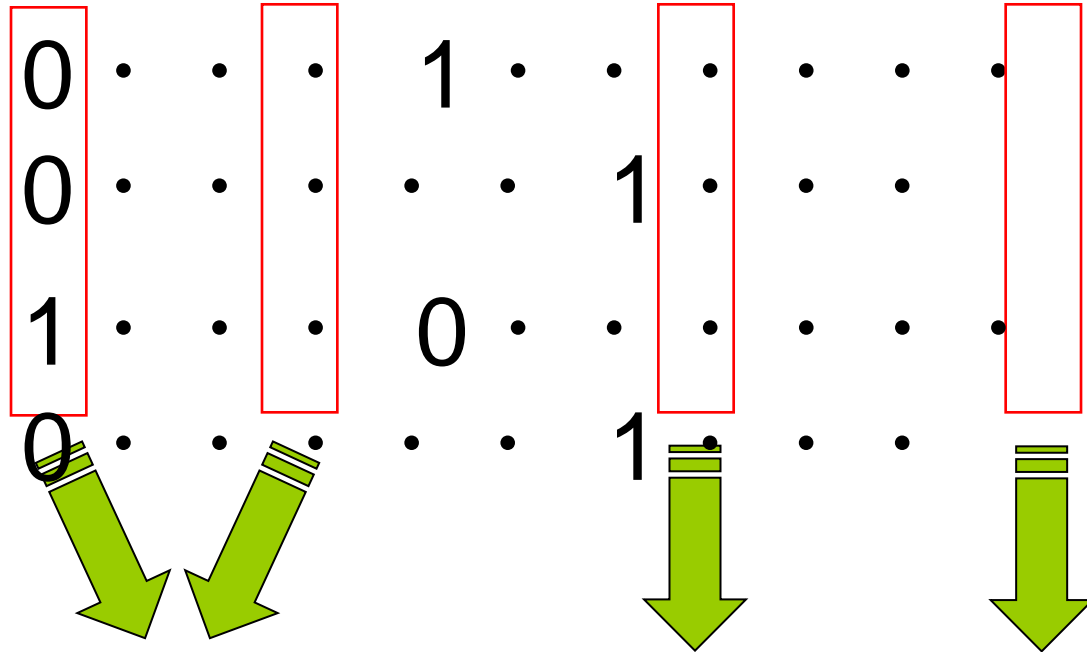
# c-Secure Codes



# Marking Assumption

[Boneh-Shaw 1995]

“undetectable positions”



0 or 1 or ‘?’

0                      1



# Preceding Results

- Tardos' s  $c$ -secure code [2003]
  - code lengths of optimal order
  - a variant has length asymptotically  $1/19$  of Tardos [Nuida et al. 2009]
- The shorter, the better
- Further shorter code?
  - e.g. for restricted number  $c$  of pirates?
    - 2-secure, 3-secure, ...

# Single or Joint Decoder

- Tracing in Tardos code uses a score for **individual** user
- Some preceding 2- or 3-secure codes use “**parent search**” technique
  - Search for a **group of users** whose codewords can generate the attack word
- More powerful, but less speedy and more difficult to evaluate theoretically

# Our Result

- Short 3-secure code with security proof
- Codeword generation is not new
- Tracing algorithm consists of 2 parts
  - 1st part: Score calculation phase
    - Defying “unbalanced” attack strategy
  - 2nd part: Parent search phase
    - Defying “balanced” attack strategy
  - Making the security proof less complex



# Codeword Generation

- Each bit of each codeword is chosen uniformly at random
  - Same as Tardos code, but with no bias
- The case of probability  $p \neq 0.5$  to choose '1' is also analyzed
- According to the present evaluation,  $p = 0.5$  minimizes the “main term” of error probability

# Tracing – 1st Phase

- For each codeword  $w$ , Calculate “(code length) – (Hamming distance of  $w$  and the attack word)” as score of the user
- Then a user is accused, if the score exceeds a suitably chosen threshold
  - If attack strategy is “unbalanced”, then the success probability of this phase becomes higher

# Feasible Sets & Parents

- $F(w_1, w_2, w_3) := \{\text{attack words which can be generated by } w_1, w_2 \text{ and } w_3\}$
- $T(y) := \{ \{u_1, u_2, u_3\} \mid y \text{ in } F(w_1, w_2, w_3) \}$
- Note: {the 3 pirates} is in  $T(\text{attack word})$

# Tracing – 2nd Phase

- If  $\mathcal{T}' = \{T \in \mathcal{T}(y) \mid T \cap T' \neq \emptyset \forall T' \in \mathcal{T}(y)\}$  is empty, then output nobody
- If  $\bigcap \mathcal{T}'$  is non-empty, then output its members
- Otherwise, at least one pirate is determined with high probability, by checking the “shape” of  $\mathcal{T}'$ 
  - Thanks to its “asymmetry” (see below)

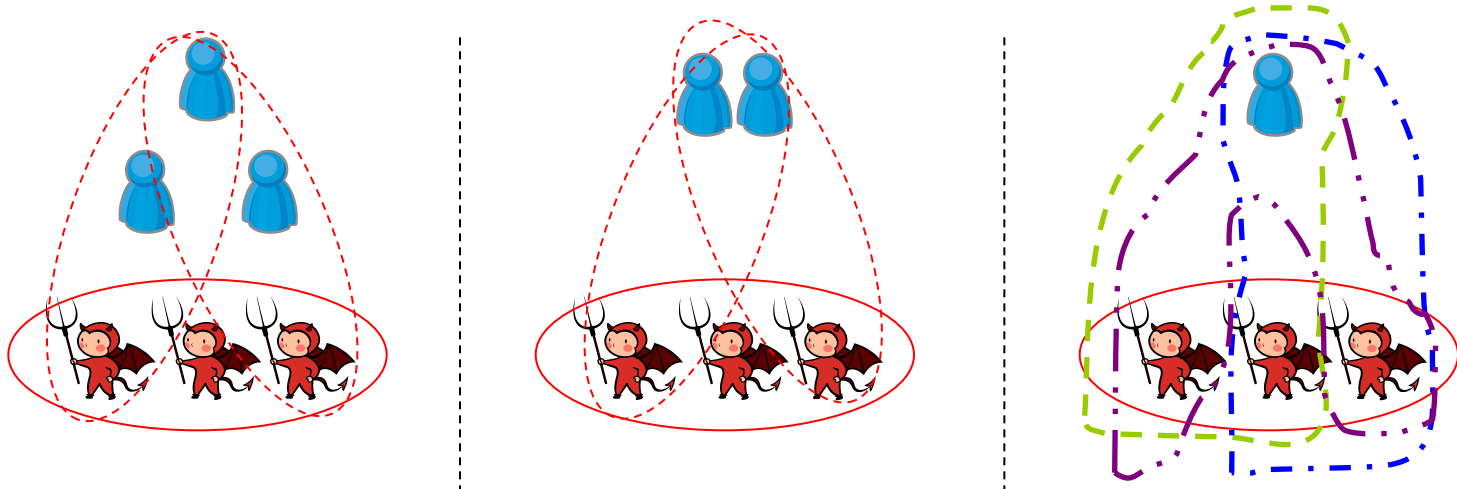
# Some Intuition for 2nd Phase (1)

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- The first case is rare, if the code length is sufficiently large
- When the attack strategy is “balanced”, the second step is likely to output some pirate (and no innocent)

# Some Intuition for 2nd Phase (2)

- The last step fails only when the following “symmetric” pattern occurs
  - Its probability is negligible, by our analysis



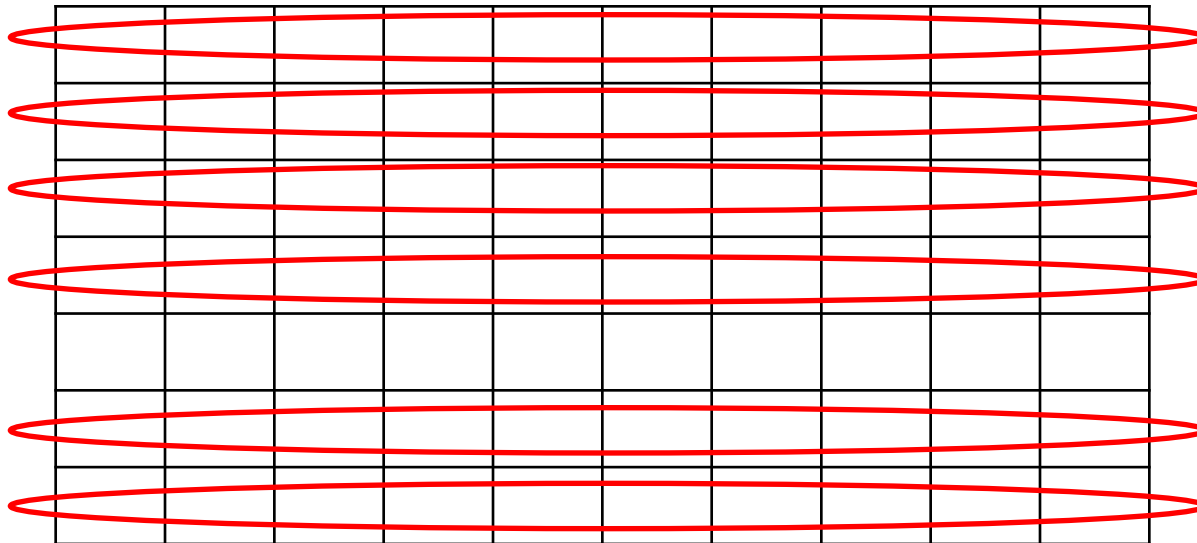
# Performance Evaluation

- We gave a formula of error probability
  - The “main term” is about  $N^3(7/8)^m / 6$
- Example of code lengths

|                   |           |              |              |
|-------------------|-----------|--------------|--------------|
| user number N     |           | 300          | 1e+6         |
| error probability |           | 1e-11        | 1e-3         |
| code length       | Nuida '09 | 1309         | 877          |
|                   | ours      | <b>420</b>   | <b>349</b>   |
| ratio             |           | <b>32.1%</b> | <b>39.8%</b> |

# Observation for Speedup (1)

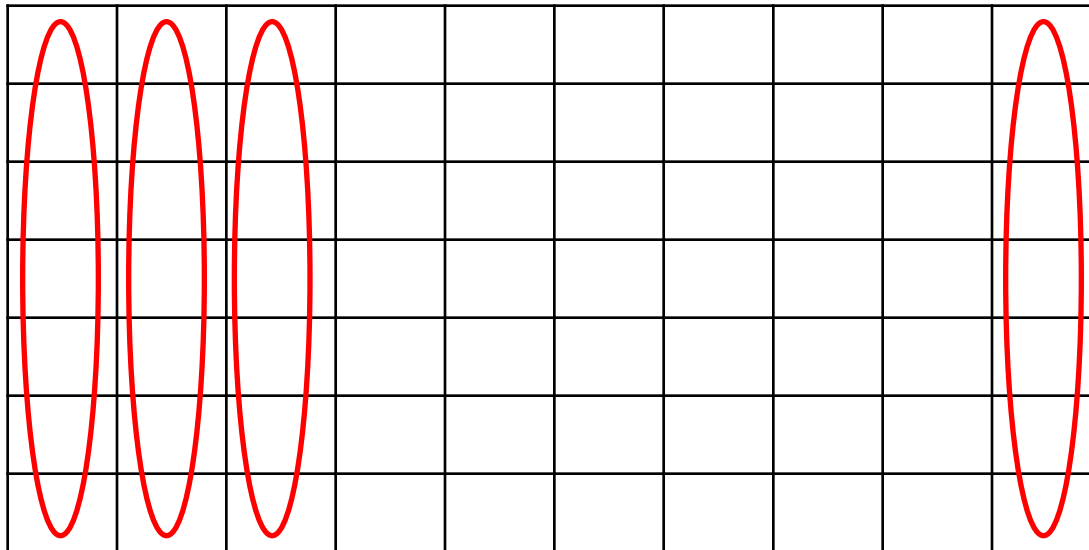
- In a naïve calculation of the set  $T(y)$ , each row of the codeword matrix is evaluated about  $N^2/2$  times





# Observation for Speedup (2)

- I tried to evaluate the codeword matrix column-wise, instead of row-wise, to avoid the duplicated evaluation
  - Detailed analysis is future work



# Conclusion

- We constructed short 3-secure code, with pirate tracing algorithm combining Tardos's score calculation method with parent search (joint decoding) method
- The code lengths are about 30% to 40% shorter than the existing shortest 3-secure codes
- Speedup of tracing is future work